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HELICAL EDDIES OF DELTA WINGS

Robert Legendre

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HELICAL EDDIES OF DELTA WINGS

Note (x) by Robert Legendre, presented by Maurice Roy

The problem is reduced to the solution of a nonlinear integrodifferential equation.

1. If, in accordance with the scheme proposed by Maurice Roy, the flow in the vicinity of the apex of a delta wing with helical eddy bands is quasi-irrotational and conical and if, either because the sweepback angle f is high or the Mach number approaches unity, the potential φ is more or less an harmonic function of y and z (Oxyz orthogonal axes, O apex of wing, Ox axis of wing, Oy in plane of wing), this potential is approximately the real part of /3814

$$\Phi = V_0 x \cos \alpha + V_0 x \sin \alpha \cot F(\zeta), \quad \zeta = \frac{y + iz}{x \cot f},$$

where α is the incidence and V_0 the velocity at infinity.

The components u, v, w of the velocity are given by

$$\frac{u}{V_0} = \cos \alpha + \sin \alpha \cot f \, \mathcal{R}(F - \zeta F_\zeta), \quad \frac{v - iw}{V_0} = \sin \alpha \, F_\zeta .$$

2. The function $F(\zeta)$ is holomorphic in the plane ζ cut by the trace of the wing, $-1 < \zeta < 1$, prolonged by the traces of the two bands, except at infinity, in the vicinity of which it behaves as $-i\zeta$ with $F - \zeta F_\zeta = 0$, so that

[redacted]

$$u = V_0 \cos \alpha, \quad v = 0, \quad w = V_0 \sin \alpha.$$

In the plane θ defined by $\zeta = \cos \theta$, the function $F(\cos \theta)$ is holomorphic, except on the traces of the bands and at infinity. It is real for real θ and for the purely imaginary $\theta = (2n + 1)\,\pi/2$, where n is an integer.

The function $F_\zeta(\zeta) = F_\zeta(\cos \theta)$ is zero in $\theta = (2n + 1)\pi/2$ and finite for $\theta = n\pi$, this last condition so that Joukowsky's rule may be satisfied at the leading edges.

3. The continuity of pressure across a band imposes the condition of continuity of the intensity V of the velocity, that is, within the framework of the linearization which already justifies the form of Φ, and with the proviso that the discontinuities in v and w are moderate, the continuity of the component u or the real part of the function $F - \zeta F_\zeta$. This function can now be represented by a distribution of sources over the traces of the bands in the plane θ.

$$F - \zeta F_\zeta = \int_0^\infty \ln\,[(\sin \sigma - \sin \theta)(\sin \sigma - \sin \theta)]\; D_\gamma(\gamma)\, d\gamma,$$

where γ is a parameter describing the band attached to the leading edge $\theta = 0$. This parameter increases from 0 at the leading edge to infinity at the end of the band. The function $\sigma(\gamma)$ is a parametric /3815
representation of the band $\theta = \sigma(\gamma)$, and $D(\gamma)$ is a real function.

4. The above representation of $F - \zeta F_\zeta$ can be integrated:

$$F = \int_0^\infty \ln\,[(\sin \sigma - \sin \theta)(\sin \bar{\sigma} - \sin \theta)]\; D_\gamma(\gamma)\, d\gamma$$

$$- \int_0^\infty \frac{\cos\theta}{\cos\sigma} \ln \left[\frac{\sin \frac{\sigma - \theta}{2}}{\cos \frac{\sigma + \theta}{2}} \right] D_\gamma(\gamma) d\gamma - \int_0^\infty \frac{\cos\theta}{\cos\bar{\sigma}} \ln \left[\frac{\sin \frac{\bar{\sigma} - \theta}{2}}{\cos \frac{\bar{\sigma} + \theta}{2}} \right] D_\gamma(\gamma)\, d\gamma,$$

$$F_\zeta = - \int_0^\infty \ln \left[\frac{\sin \frac{\sigma - \theta}{2}}{\cos \frac{\sigma + \theta}{2}} \right] \frac{D_\gamma(\gamma) d\gamma}{\cos\sigma} - \int_0^\infty \ln \left[\frac{\sin \frac{\bar{\sigma} - \theta}{2}}{\cos \frac{\bar{\sigma} + \theta}{2}} \right] \frac{D_\gamma(\gamma) d\gamma}{\cos\bar{\sigma}} .$$

The values of the logarithms are zero for $\theta = - \pi/2$ in F_ζ. That of the first logarithithm in F is arbitrary.

5. In order that the function $F - \zeta F_\zeta$ be zero at infinity, the sum of the sources must be zero:

(1) $$D(0) = D(\infty) = 0.$$

In order that F_ζ, zero in $\theta = - \pi/2$, should also be zero in $\theta = = + \pi/2$, we must have

(2) $$\int_0^\infty \mathcal{R}\left(\frac{1}{\cos\sigma}\right) D_\gamma(\gamma)\, d\gamma = 0.$$

There is no condition to be written at the leading edge, where F is finite at least if $D(\gamma)$ is regular.

In order that $F(\zeta) = - i$ at infinity, we must have

$$(3) \qquad -\int_0^\infty \mathcal{R}\left(\frac{\sigma}{\cos\sigma}\right) D_\gamma(\gamma)\, d\gamma = 1.$$

6. The traces of the flow surfaces in the plane ζ are defined by:

$$x\, d\zeta = \tan f(d\gamma + idz) - \zeta\, dx = [\tan\,(v + iw) - \zeta u]\, dt$$

$$= V_0 \cos\alpha[\tan\alpha \tan f \bar{F}_{\bar{\zeta}} - \zeta(1 + \tan\alpha \cot f\, \mathcal{R}(F - \zeta F_\zeta))]\, dt,$$

where t is time. Substitution of the parameter γ gives:

$$\mathcal{R}\left[\frac{\tan\alpha \tan f\, \bar{F}_{\bar{\zeta}} - \zeta(1 + \tan\alpha \cot f\, \mathcal{R}(F - \zeta F_\zeta))}{i\zeta_\gamma}\right] = 0.$$

In particular, the helical bands are flow surfaces $\zeta = \xi(\gamma) = \cos[\sigma(\gamma)]$ for the flow on both sides corresponding to the values $F \pm \delta F/2$ of the function F. This implies the two conditions:

$$\zeta = \xi(\gamma) \begin{cases} (4) \quad \mathcal{R}\left[\dfrac{\delta \bar{F}_{\bar{\zeta}}}{i\zeta_\gamma}\right] = 0 \\[2ex] (5) \quad \mathcal{R}\left[\dfrac{\tan\alpha \tan f\, F_{\bar{\zeta}} - \zeta[1 + \tan\alpha \cot f\, \mathcal{R}(F - \zeta F_\zeta)]}{i\zeta_\gamma}\right] = 0. \end{cases}$$

7. Condition (4) makes it possible to calculate $D_\gamma(\gamma)$ as a /3816

function of $\sigma(\gamma)$

$$\delta\bar{F}_{\bar{\zeta}} = - i\pi \int_{\gamma}^{\infty} \frac{D_{\gamma}(\gamma)\, d\gamma}{\bar{\xi}} = - \pi\lambda(\gamma)\xi_{\gamma},$$

$$D_{\zeta}(\gamma) = i\bar{\xi} \frac{\partial}{\partial\gamma} [\lambda\xi_{\gamma}],$$

where λ is a real function defined by

$$\ln \frac{\lambda}{\lambda_0} = - \int_0^{\gamma} \frac{\bar{\xi}\cdot\xi_{\gamma\gamma} + \xi\cdot\xi_{\gamma\gamma}}{\bar{\xi}.\xi_{\gamma} + \xi.\bar{\xi}_{\gamma}}\, d\gamma.$$

The constant λ_0 is found from condition (3) and conditions (1) and

(2) are global for $\sigma(\gamma)$. Other conditions to be fulfilled for λ to be acceptable relate to the behavior of $\sigma(\gamma)$ in the neighborhood of the zeros of

$$\frac{\partial}{\partial\gamma} (\xi\cdot\bar{\xi}) = \bar{\xi}\cdot\xi_{\gamma} + \xi\cdot\bar{\xi}_{\gamma}.$$

8. The function $\xi(\gamma) = \cos [\sigma(\gamma)]$ which remains to be determined can be linked with the real function $k(\gamma)$

$$\sigma(\gamma) = (k + ik_{\gamma})e^{i\gamma}, \qquad \sigma_{\gamma} = (k + k_{\gamma\gamma})e^{i(\gamma + \frac{\pi}{2})}.$$

The parameter γ implicitly chosen is such that $\gamma + \pi/2$ is the angle made with the real axis by the tangent to the trace of the helical eddy band in the plane θ, whereas the radius of curvature R of this band is $R = k + k_{\gamma\gamma}$.

Thus, condition (5) is a nonlinear integrodifferential equation applicable to the unique real function $k(\gamma)$, which must also satisfy the global conditions of paragraph 7 defining the constants of integration.

9. It is difficult to calculate the solution $k(\gamma)$ or even to show that a well-defined one exists. If it does, F is a function of ζ, α, f. However, if $\tan\alpha \cot f$ is small compared with 1, condition (5) reduces to

$$\text{(5a)} \qquad \mathcal{R}\left[\frac{\tan\alpha \tan f \, \bar{F}_{\xi} - \zeta}{i\zeta_{\gamma}} \right] = 0$$

and F depends only on ζ and $\tan\alpha \tan f$.

10. The experimental results suggest the choice for $k(\gamma)$ of an even function of γ, zero for $\gamma = 0$ and such that the radius $R = k + k_{\gamma\gamma}$ quickly tends to zero as γ increases. If $k(\gamma)$ depends on a fairly large number of parameters, to be determined as a function of α and f, the global conditions can be satisfied, and condition (5) or (5a) can be written at several points to obtain an approximation.

11. In a previous paper [1] relating to the flow of an incompressible fluid around a delta wing, the equation satisfied by the potential and the condition of continuity of pressure were not
simplified, but the discussion was more complex. No hypothesis was /3817
made concerning the order of magnitude of the velocity, which could assume a high or even infinite value at the end of a band, as suggested by the experimental results, complicated though these may be, on the effects of viscosity.

(x) Meeting of 4 December 1963

[1] Helical Eddy Bands at the Leading Edges of a Delta Wing, La Recherche aeronautique, May 1959.

Some Remarks on the Above Note

Maurice Roy

I take all the more pleasure in presenting the above Note in that it makes an important contribution to the determination, for a plane indeterminate delta wing, of the helical eddy bands, which I introduced in 1952 [1] to schematize the flow around a wing of this sort.

This very remarkable advance consists, thanks to an ingenious choice of the parameter γ and the treatment of the function $(F - \zeta F_\zeta)$, in reducing the determination in question to the solution of a unique and real function $K(\gamma)$ satisfying a real and nonlinear integrodifferential equation.

This concept confirms that of the equivalence of a helical eddy band, for R. Legendre's reduced transverse pseudo-flow in the plane ζ, and a distribution of source eddies over the "trace" of the band, a distribution reducible to a simple source distribution for the determination of Legendre's function $(F - \zeta F_\zeta)$.

However, I would like to recall that, in accordance with the concept that I defined in 1957 [2], this band would be bordered by a "flange" type of source eddy, that is, one of finite intensity distributed over a fairly small cross-sectional area and represented schematically, in relation to the flow close to the wing, by a concentrated source eddy, a so-called "apex eddy." In fact, the sources in question are negative, i.e., the above eddies are of the well type. In the transverse pseudo-flow (plane η, ζ of my 1957 Note), these wells are compensated by distributed sources, which might be schematized, in relation to the flow close to the wing, by one or more finite, discrete sources.

It is to be hoped that the ingenious theoretical simplification of the above Note will soon produce some numerical solutions for special cases.

[1] Comptes rendus, 234, p. 2501, 1952.
[2] Comptes rendus, 244, p. 1105, 1957.

FARADAY TRANSLATIONS
15 PARK ROW
NEW YORK, N.Y. 10038